Calculation Policy

Background

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what children should achieve. In addition, there is a whole new assessment method, as the removal of levels gives schools greater freedom to develop and use their own systems. One of the key differences is the level of detail included, indicating what children should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered. For example, in Year 2, it is suggested that children should be able to 'add and subtract one-digit and two-digit numbers to 20, including zero' and a few years later, in Year 5, they should be able to 'add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)'. In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils' calculation skills. However, the expectation of using formal methods is rightly coupled with the explicit requirement for children to use concrete materials and create pictorial representations – a key component of the mastery approach. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (*reasoning*). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. A list of terminology is located at Appendix A to this document.

How to use the policy

This mathematics policy is a guide for all staff at Springfield Primary Academy based on the guidance from White Rose. It is purposely set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the **focus must always remain on breadth and depth rather than accelerating through concepts.** Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems. All teachers use the White Rose Schemes of Learning basing their planning around their year group blocks in line with the yearly overviews and not to move onto a higher year groups scheme of learning. These modules use the Singapore Maths Methods and are affiliated to the workings of the 2014 Maths Programme of Study.

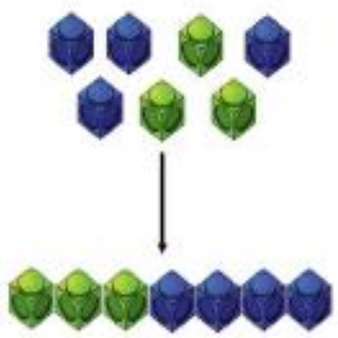
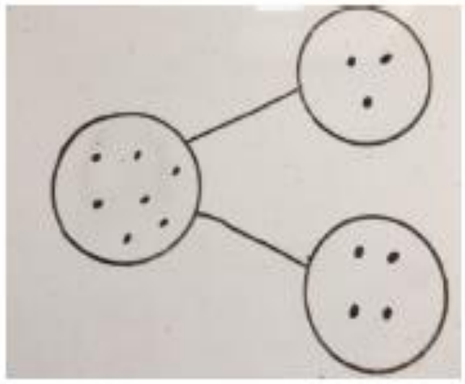
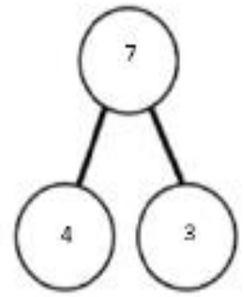
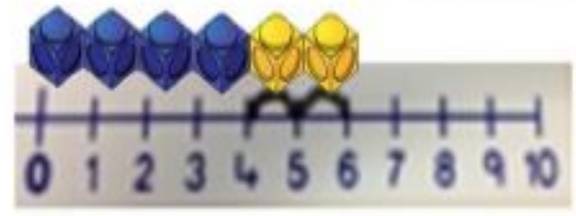
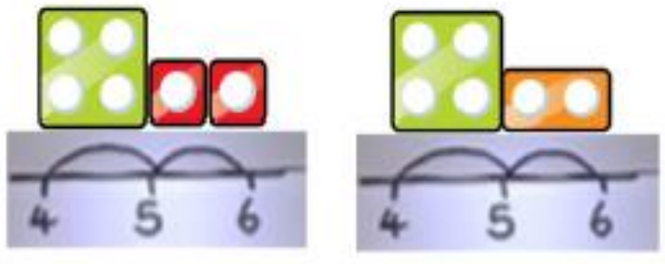
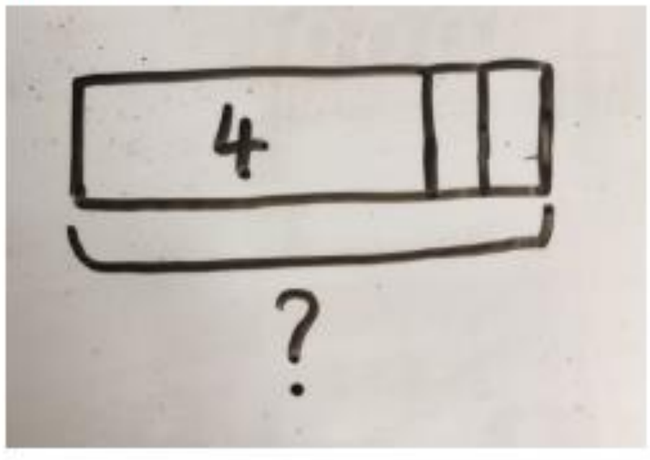
Teachers are expected to use concrete resources and pictorial representations in line with White Rose Maths Small Steps Guidance. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases during their journey through school.

Content of the Policy

> The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.
>
> *2014 Maths Programme of Study*

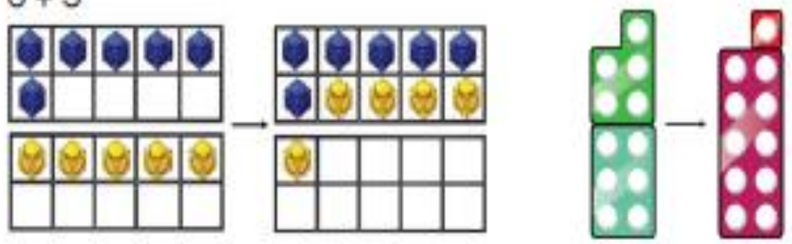
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

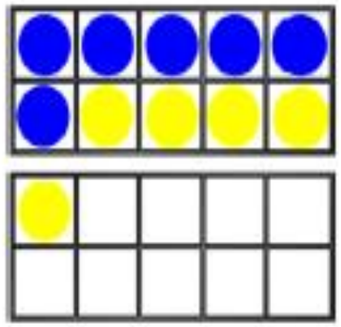
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

6 + 5



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

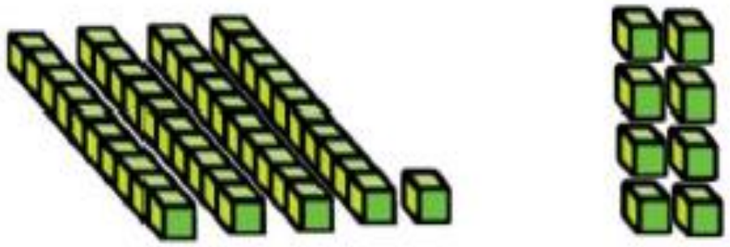
$6 + \square = 11$

$6 + 5 = 5 + \square$

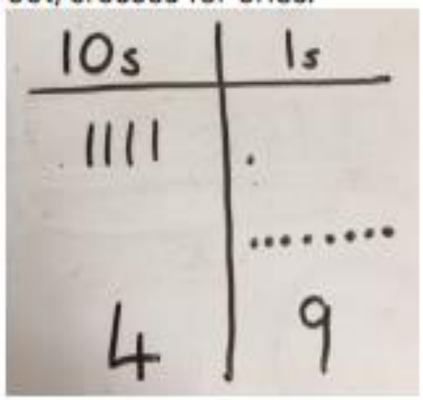
$6 + 5 = \square + 4$

TO + O using base 10. Continue to develop understanding of partitioning and place value.

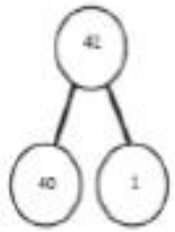
41 + 8



Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



41 + 8



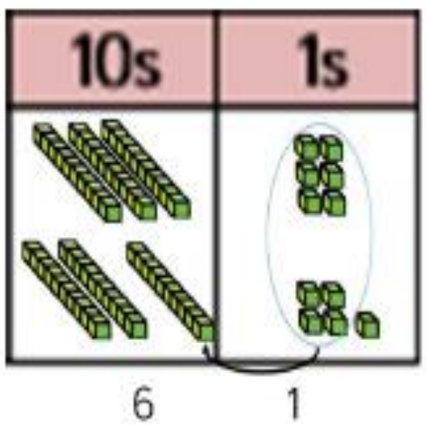
1 + 8 = 9

40 + 9 = 49

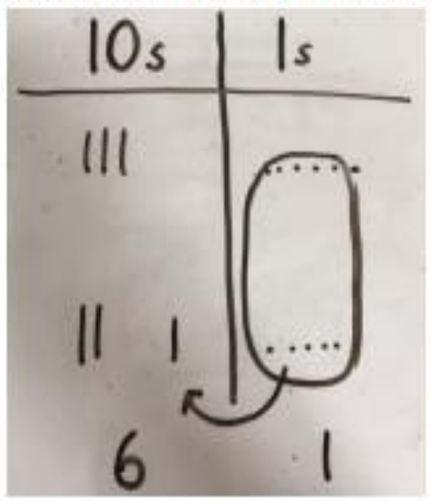
	4	1
+		8
	4	9

TO + TO using base 10. Continue to develop understanding of partitioning and place value.

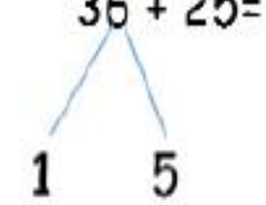
36 + 25



Chidlren to represent the base 10 in a place value chart.



Looking for ways to make 10.



30 + 20 = 50

5 + 5 = 10

50 + 10 + 1 = 61

Formal method:

36

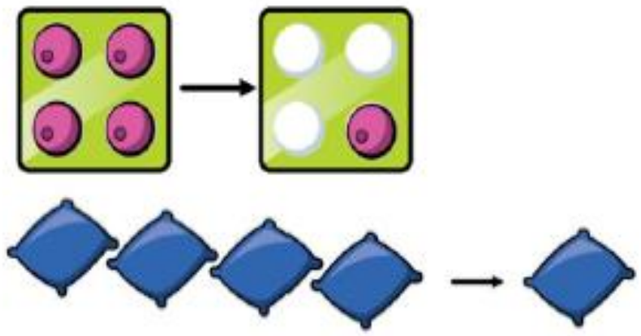
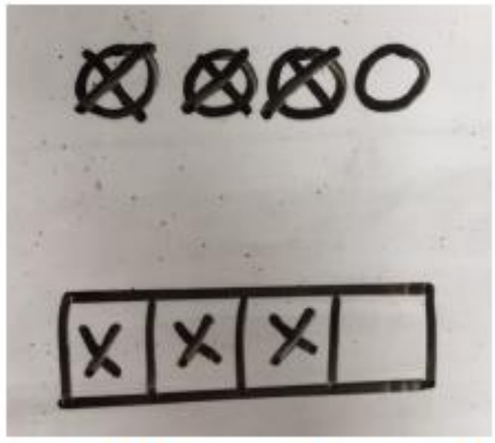
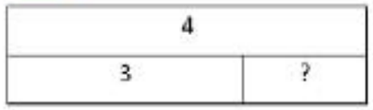
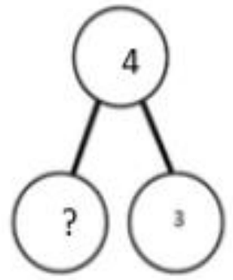
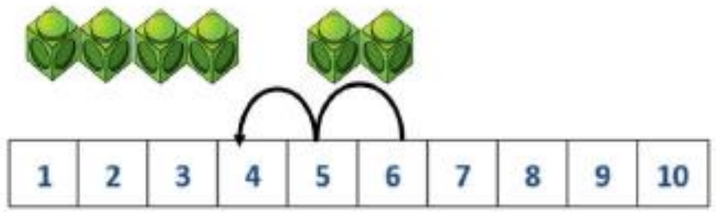
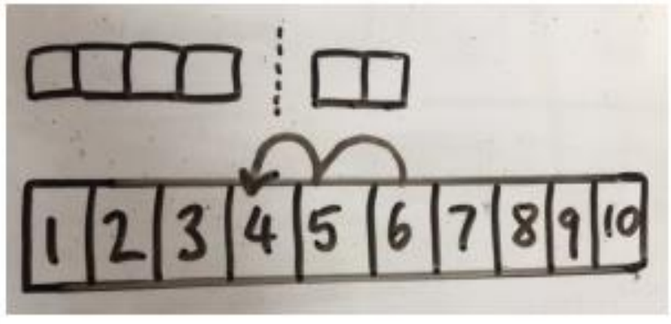
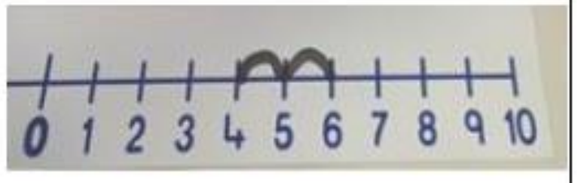
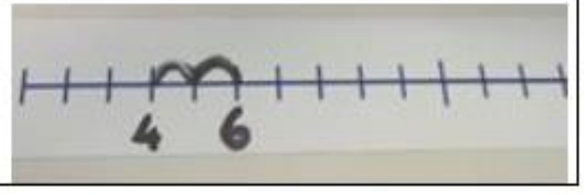
+25

61

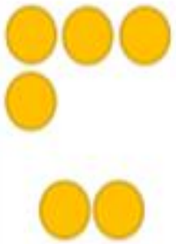
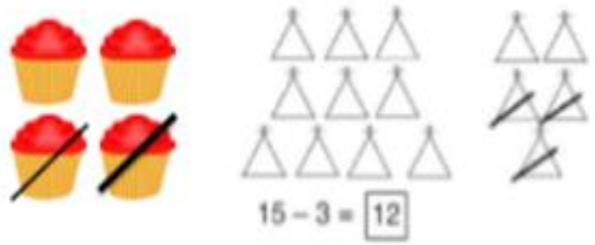
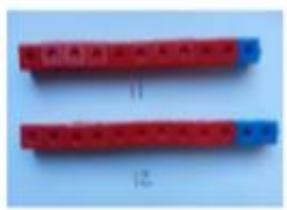
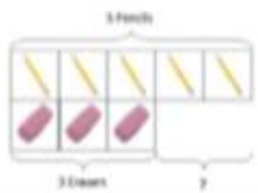
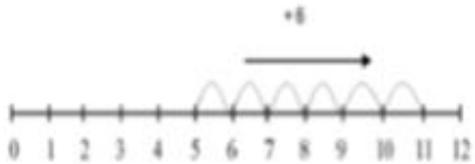
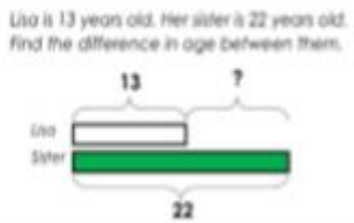
1

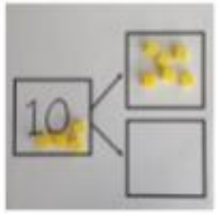
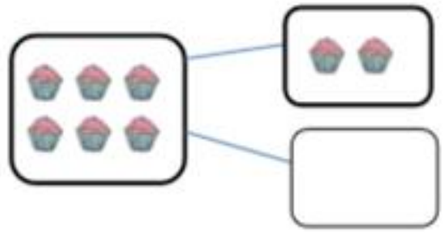
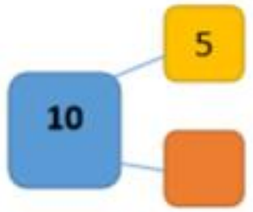
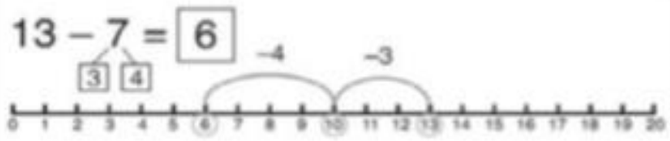
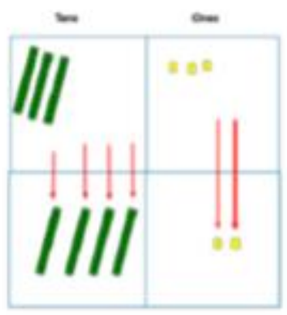
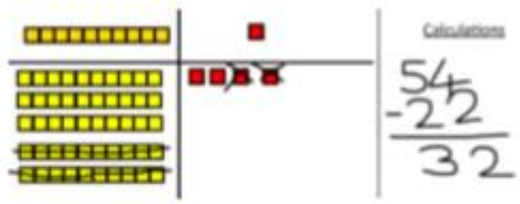
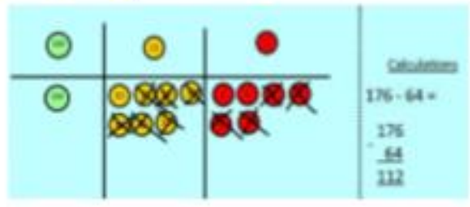
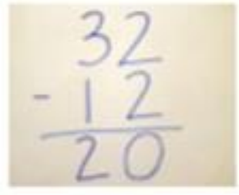
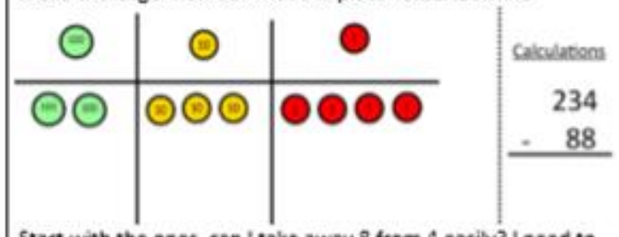
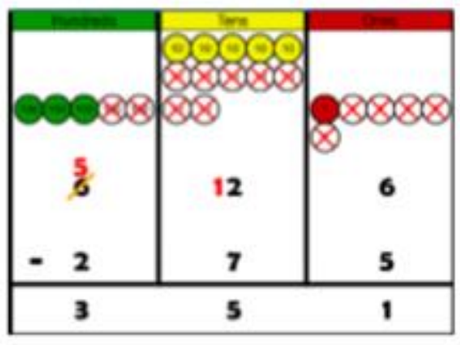
Calculation policy: Subtraction

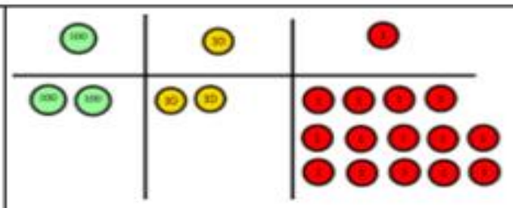
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 – 3  
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4  	Children to represent what they see pictorially e.g.  	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

Subtraction

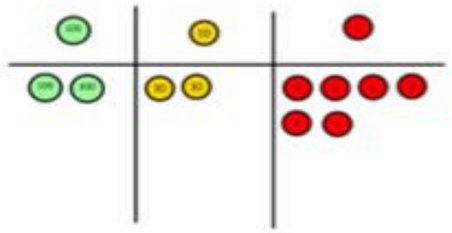
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  **6 – 2 = 4**	Cross out drawn objects to show what has been taken away. 	**18 -3= 15** **8 – 2 = 6**
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. **13 – 4**  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference.	 Count on to find the difference. **Comparison Bar Models** Draw bars to find the difference between 2 numbers. 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? **10 - 6 =**	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	**14 – 9 =**  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	**16 – 8=** How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.	Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	$47-24=23$ $40+7$ $-\ 20+4$ $20+3$ This will lead to a clear written column subtraction. 
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. 	$600 \quad 110 \quad 16$ $\cancel{700} \quad \cancel{20} \quad \cancel{6}$ $-\ 300 \quad 50 \quad 8$ $300 \quad 60 \quad 8$ Children can start their formal written method by partitioning the number into clear place value columns.



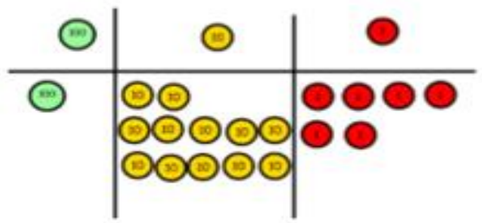
Calculations
234
- 88

Now I can subtract my ones.



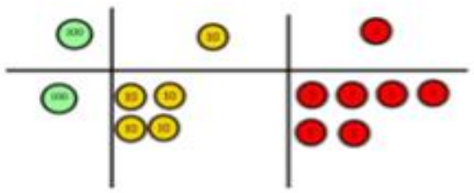
Calculations
234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Calculations
234
- 88

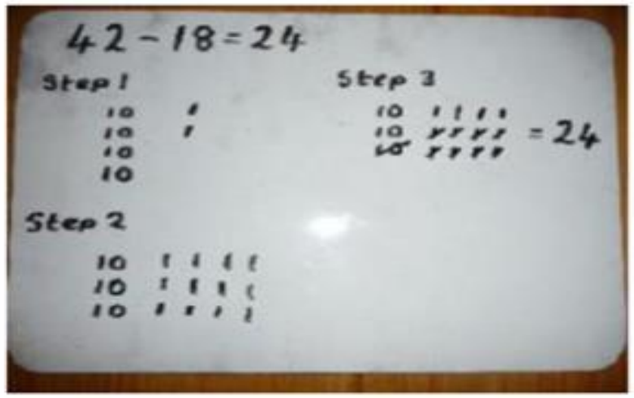
Now I can take away eight tens and complete my subtraction



Calculations
~~2~~ ~~3~~ 4
- 88
146

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and

When confident, children can find their own way to record the exchange/regrouping.



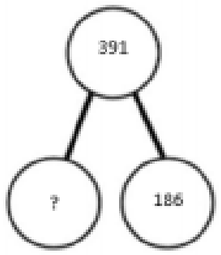
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

$$\begin{array}{rccc} & 6 & 11 & 16 \\ & \cancel{7} & \cancel{2} & \cancel{6} \\ - & 3 & 5 & 8 \\ \hline & 3 & 6 & 8 \end{array}$$

Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

□ = 391 – 186

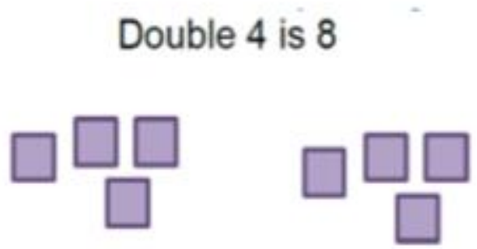
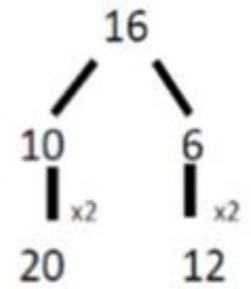
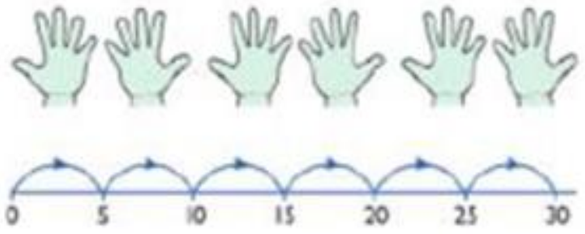
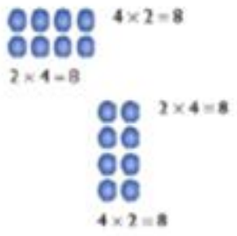
391
-186
___

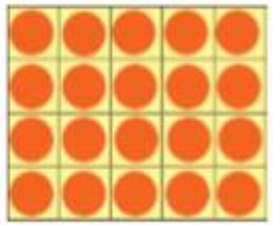
What is 186 less than 391?

Missing digit calculations

$$\begin{array}{rccc} & 3 & 9 & \square \\ - & \square & \square & 6 \\ \hline & \square & 0 & 5 \end{array}$$

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	Partition a number and then double each part before recombining it back together. 
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. **2, 4, 6, 8, 10** **5, 10, 15, 20, 25, 30**
Repeated addition	  Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences. 	 Draw arrays in different rotations to find **commutative** multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition.



Link arrays to area of rectangles.

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

5 x 3 = 15

3 x 5 = 15

Grid Method

Show the link with arrays to first introduce the grid method.

x	10	3
4		

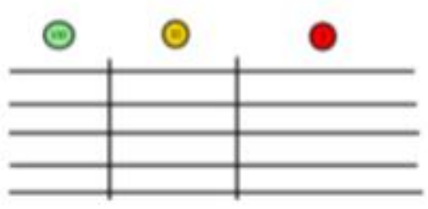
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

x	T	U

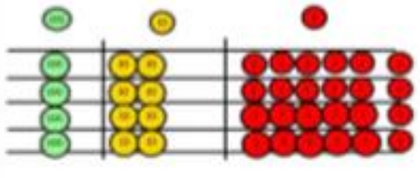
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Calculations
4 x 126

Fill each row with 126.

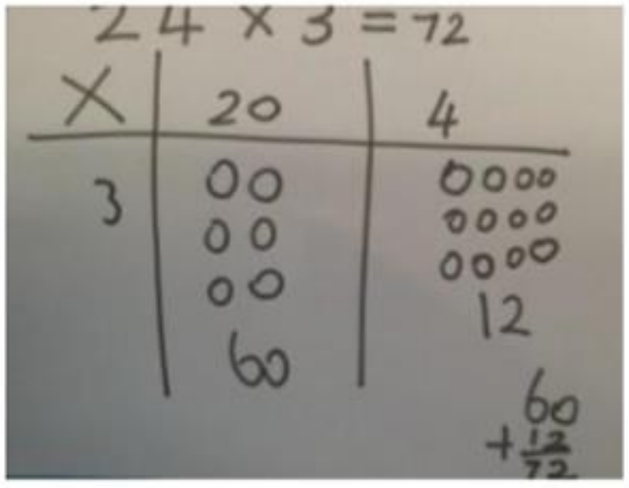


Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	20	3
4	80	12

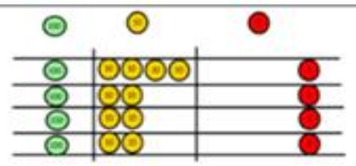
= 92

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

×	10	6	
40	400	240	= 640
8	80	48	= 128
			768

×	200	50	3
6	1200	300	18

= 1518

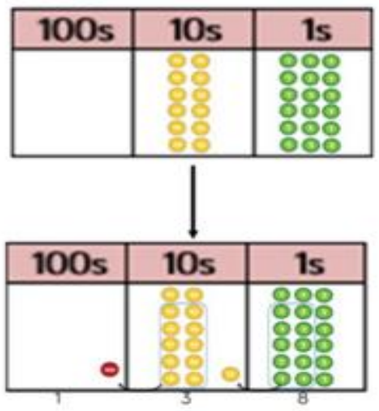


Then you have your answer.

Column multiplication

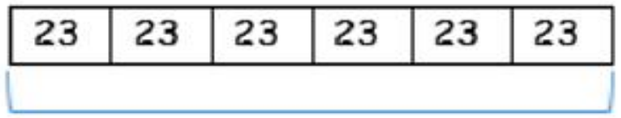
Children can continue to be supported by place value counters at the stage of multiplication.

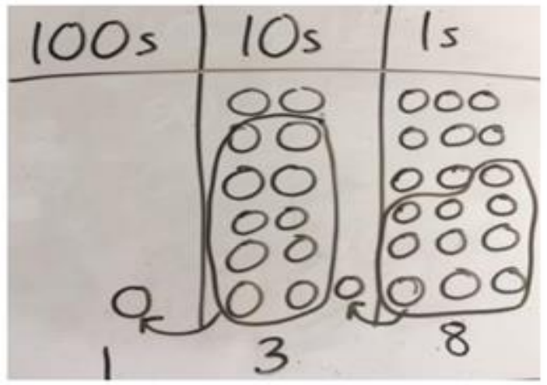
6 x 23



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

$$\begin{array}{r} 253 \\ \times\ \ 6 \\ \hline 18 \\ 300 \\ +\ 1200 \\ \hline 1518 \end{array}$$

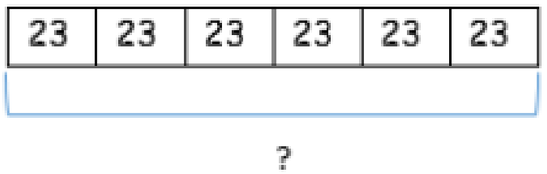
6 × 3
6 × 50
6 × 200

$$\begin{array}{r} 48 \\ \times\ 16 \\ \hline 288 \\ 480 \\ 1 \\ \hline 768 \end{array}$$

$$\begin{array}{r} 435 \\ \times\ \ 8 \\ \hline 3480 \end{array}$$

$$\begin{array}{r} 456 \\ \times\ 38 \\ \hline 3648 \\ 13680 \\ \hline 17328 \end{array}$$

Conceptual variation; different ways to ask children to solve 6 × 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

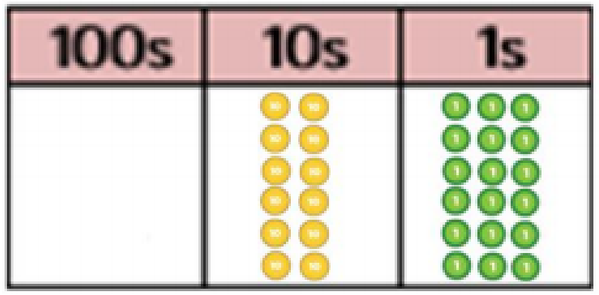
Find the product of 6 and 23

6 × 23 =

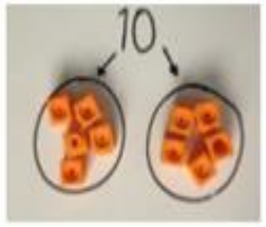
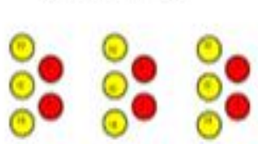
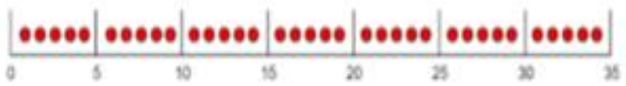
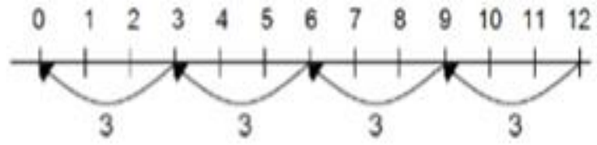
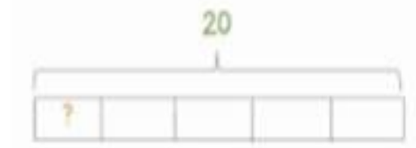
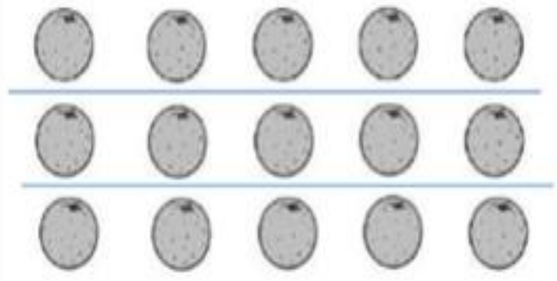
☐ = 6 × 23

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ \ 6 \\ \hline \\ \hline \end{array}$$

What is the calculation?
What is the product?



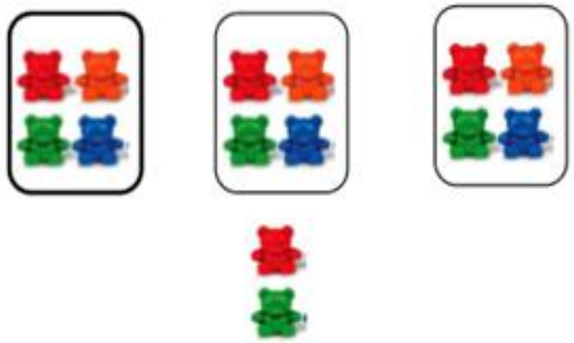
Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	**Share 9 buns between three people.** **$9 \div 3 = 3$**
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  $96 \div 3 = 32$   	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  $20 \div 5 = ?$ $5 \times ? = 20$	**$28 \div 7 = 4$** **Divide 28 into 7 groups. How many are in each group?**
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. **$7 \times 4 = 28$** **$4 \times 7 = 28$** **$28 \div 7 = 4$** **$28 \div 4 = 7$**

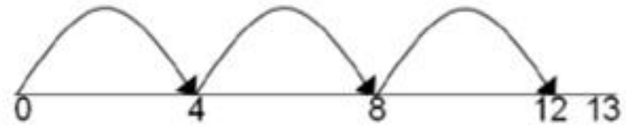
Division with a remainder

14 ÷ 3 =

Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.

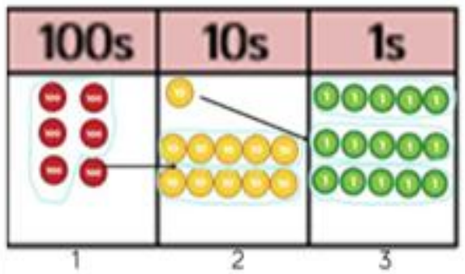


Draw dots and group them to divide an amount and clearly show a remainder.

Complete written divisions and show the remainder using r.

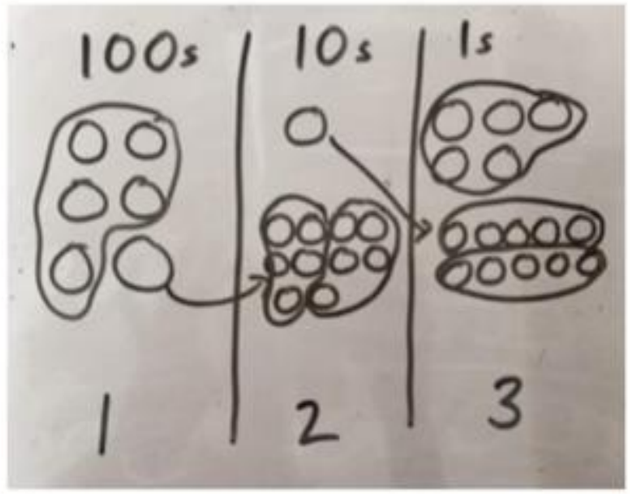
Short division

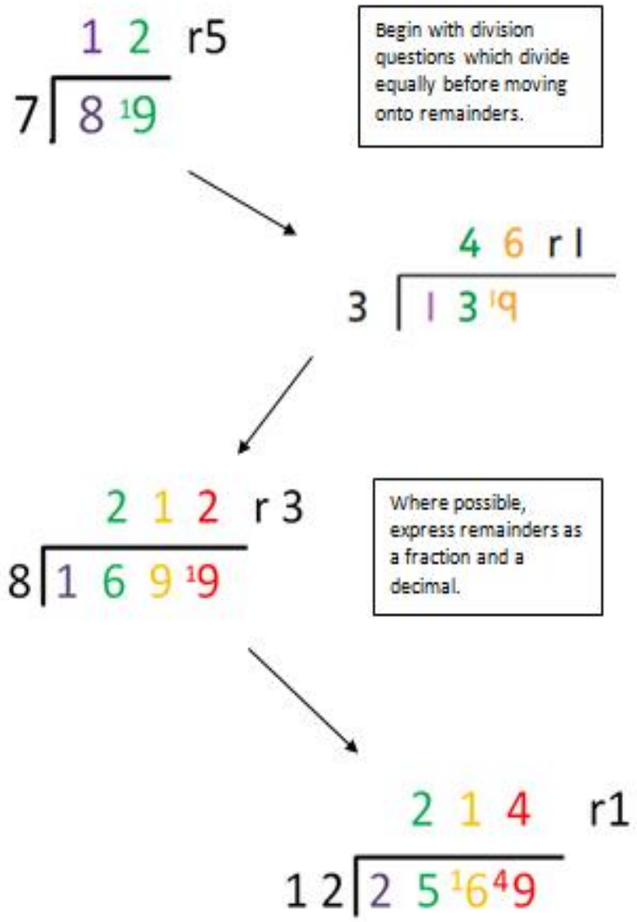
615 ÷ 5



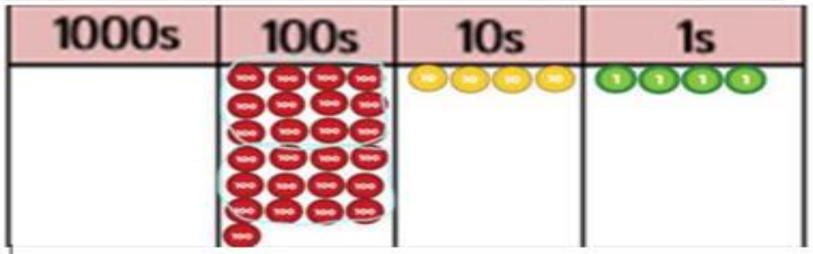
1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



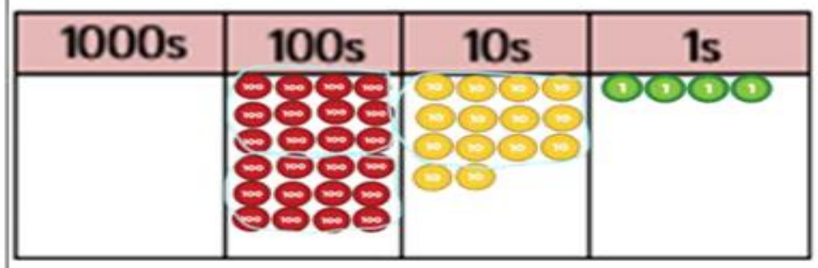


We can't group 2 thousands into groups of 12 so exchange them for 20 hundreds.



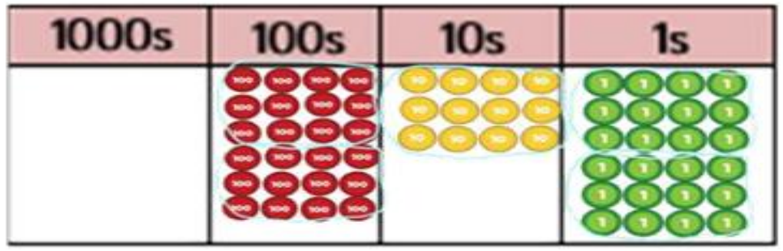
We can group 24 hundreds into groups of 12 which leaves us with 1 hundred

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no reminders.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$5\overline{)615}$

615 ÷ 5 =

☐ = 615 ÷ 5

What is the calculation?
What is the answer?

Overview for year groups

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Mathematical Language

High expectations of the mathematical language used are essential, with staff only accepting what is correct. Consistency across the school is key:

Correct Terminology	Incorrect Terminology
ones	units
is equal to (is the same as)	equals
exchange exchanging regrouping	stealing borrowing
calculation equation	generic term of 'sum' or 'number sentence'
bar model	
known unknown	
part whole	